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Portrait of the string as a random walk

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Abstract: In this paper we analyze a Rutherford type experiment where light probes are inelastically scattered by an ensemble of excited closed strings, and use the corresponding cross section to extract density-density correlators between different pieces of the target string. We find a wide dynamical range where the space-time evolution of typical highly excited closed strings is accurately described as a convolution of brownian motions. Moreover, we show that if we want to obtain the same cross section by coherently scattering probes off a classical background, then this background has to be time-dependent and singular. This provides an example where singularities arise, not as a result of strong gravitational self-interactions, but as a byproduct of the decoherence implicit in effectively describing the string degrees of freedom as a classical background.

Keywords: Bosonic Strings, Black Holes in String Theory.

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1. Introduction

The idea that highly excited fundamental strings behave like random walks is not a new one. It was introduced as a physically sensible assumption in [1], where a model to describe a gas of strings at very high density was proposed. Namely, it was assumed that at fixed time a long string resembles the path of a brownian motion, so that the average distance between two points separated by a length l of string is proportional to $\sqrt{l}$. This idea is supported by the dependence of the mean square radius of the string on its mass, $\langle r^2 \rangle \propto M$, which is assumed to be proportional to the length. This was first obtained by oscillator methods by the authors of [2, 3], who also applied their techniques to the study of cosmic string networks.

More recently, the random walk picture has been useful in different situations where highly excited strings are involved. It was used in [4, 5] to derive the form of the interactions in a Boltzmann equation approach to the Hagedorn gas, and in [6] to model the string-black hole phase transition that takes place at the correspondence point [7]. In fact, the use of polymer physics techniques in [6] yields a description of the string-black hole transition largely consistent with the one obtained in [8] by a thermal scalar formalism and in [9] by more conventional oscillator methods. A discussion of the relation between the thermal scalar and the counting of random walks has been recently given in [10], where the random walk is used as an intuitive geometrical picture in order to explain many features of the Hagedorn regime. Also closely related to the random walk picture is the approach where the string is actually assumed to be a composite of elementary 'string-bits' [11]–[14].

In spite of the undeniable usefulness of the idea, most of the results obtained from the random walk approach have been of a qualitative or at best semi-quantitative nature. One reason is that the relation between excited strings and random walks has never been made very precise, and it is not hard to see why. The random walk idea is basically a statement about the size and shape of highly excited strings but, as mentioned above, the only *geometric* properties actually computed for the string and compared with the random walk predictions are mean square radius [2, 3]. And obviously, a distribution is hardly characterized by its second moment alone.

Trying to refine the geometric description of a string state is tricky, since the coordinates X^μ are quantum fields on the worldsheet and, as such, undergo infinite zero-point fluctuations [15]. Indeed, even the mean square radius turns out to be infinite, and the above mentioned dependence on the mass of the string is obtained only after a subtraction is performed by hand. Attempts to obtain the higher moments $\langle r^n \rangle$ of the distribution directly by oscillator methods will involve increasingly complicated computations and an endless sequence of ambiguous, infinite subtractions.

An alternative approach is based on the observation [16] that we get infinite results because we sum aver the infinite modes of the string, whereas any real attempt to measure its shape will be limited by the time resolution ϵ of the experiment. Thus, all modes with frequency $w > 1/\epsilon$ will be averaged out and effectively cut off. In particular one can use scattering experiments, where the time resolution is related to the energy by $\epsilon \sim 1/E$. The cross sections can be used to extract form factors, and these are finite without the need for infinite subtractions and yield detailed information about the geometry of the string. This approach has been used [17, 18] to study the energy dependence of the size of string ground states, with surprising consequences regarding the behavior under Lorentz transformations and the black hole complementarity principle. It has also been used for open strings on the leading Regge trajectory [19], and very recently for typical closed strings [20].

In this paper we analyze the results of [20], where the cross section for a Rutherford type experiment with light strings scattered off highly excited closed strings was obtained. Rather than considering individual excited states, the equivalent of an 'unpolarized' cross section was computed by taking an average over all the states at a given mass level. This is exactly the kind of information that we need in order to settle the issue of the equivalence between strings and random walks, since at most one should expect an ensemble of highly excited strings to behave like some collection of random walks, and this only for some dynamical range.

This paper is organized as follows. In section 2 we present a brief review of the effective form factors computed in [20] and find the first indication of random walk behavior in their elastic part, together with a puzzle. This is solved in section 3, where the inelastic cross section is used to obtain the time dependence of density-density correlators for the target string, yielding a picture of the space-time evolution of typical strings in terms of a convolution of random walks. In section 4 we show that, if one tries to obtain the same cross sections by coherently scattering probes off a classical background, then this background has to be both time-dependent and singular. Finally, our conclusions and outlook are presented in section 5.

2. Elastic form factors and random walks

2.1 Review of string form factors

A natural way to measure the geometry of a string is by a scattering experiment. Although string amplitudes are rather complicate even for the simplest scattering processes and reflect exchanges of an infinite number of states, a simplification takes place in the Regge limit of fixed momentum transfer $t = -q^2$ and high energy E. In this region the interaction is dominated by t-channel exchange of leading Regge trajectory states. Consider for instance the Virasoro-Shapiro amplitude $\mathcal{A}(s,t)$ describing the scattering of two closed string tachyons. In the Regge limit this amplitude can be written as [21, 22]

$$\mathcal{A} \sim s^{2-q^2} \frac{\Gamma(\frac{1}{2}q^2 - 1)}{\Gamma(2 - \frac{1}{2}q^2)}, \tag{2.1}$$

where $s \sim 4E^2$ and we have set $\alpha' = 2$. At low momentum transfer the amplitude is dominated by the graviton (multiplet) pole in the gamma function and can be interpreted as the product of the high energy graviton interaction for point particles by the form factors $\mathcal{F}(q^2)$ of the scattered strings [18]:

$$\mathcal{A} \sim \frac{E^4}{q^2} (e^{-q^2 \log E})^2 = \frac{E^4}{q^2} \mathcal{F}^2(q^2). \tag{2.2}$$

The same strategy is followed in [20], where closed string tachyons[1] are used to probe a statistical ensemble of excited closed strings of mass M. An averaged interaction rate is defined by *averaging* over all initial target states at mass level N and momentum p and *summing* over all final target states at mass level N' and momentum p'. Concretely,

$$\mathcal{R}(N, N', k, k') \equiv \frac{1}{\mathcal{G}_c(N)} \sum_{\Phi_i|_N} \sum_{\Phi_f|_{N'}} |\mathcal{A}(\Phi_i, \Phi_f, k, k')|^2, \tag{2.3}$$

where the masses of the initial and final states are given by $M^2 = 2(N-1)$ and $M'^2 = 2(N' -1)$ respectively, $\mathcal{G}_c(N)$ is the degeneracy of the N^{th} closed string mass level, and k and k' are the momenta of the incoming and outgoing tachyons. In the Regge limit of high energy probes ($E \gg 1$ in string units) and fixed momentum transfer $q^2 = (k + k')^2$ the interaction rate factorizes

$$\mathcal{R}(N, N', k, k') \sim M^4 \left| \frac{\Gamma(\frac{q^2}{2} - 1)}{\Gamma(2 - \frac{q^2}{2})} (EE')^{1-\frac{q^2}{2}} \right|^2 |\mathcal{F}_{NN'}(q^2)|^2, \tag{2.4}$$

[1] Tachyons are used as probes for the sake of simplicity. As explained in [20], using other light string probes introduces polarization dependent factors in the cross section, but the target form factor is unchanged.

where $\mathcal{F}_{NN'}(q^2)$ is the *effective form factor* for the target string. For heavy targets, this is given by

$$\mathcal{F}_{NN'}(q^2) = M^{-q^2} \oint_{C_v} \frac{dv}{v} v^{N-N'} \langle V_0(-q,1) V_0(q,v) \rangle_w \,, \tag{2.5}$$

which is valid up to relative corrections of order $O(1/M)$. In this expression, V_0 is the oscillator part of the *open* string tachyon vertex operator with zero modes removed, C_v is a contour satisfying $|w| < |v| < 1$, and the correlator is evaluated on a cylinder with modular parameter $w = e^{-\beta}$, with β given by

$$\beta \approx \pi \sqrt{\frac{D-2}{6N}} = \frac{1}{2MT_H} \,, \qquad T_H = \frac{1}{2\pi} \sqrt{\frac{3}{D-2}} \,, \tag{2.6}$$

where T_H is the Hagedorn temperature in D space-time dimensions.

The interpretation of $\mathcal{F}_{NN'}(q^2)$ as an effective form factor can be motivated by noting that, for elastic scattering ($N = N'$) and $q^2 \sim 0$, eq. (2.4) becomes

$$\mathcal{R}(N, N', k, k') \sim \left| \frac{(ME)^2}{q^2} \mathcal{F}(q^2) \mathcal{F}_{NN}(q^2) \right|^2 \,, \tag{2.7}$$

where we recognize the square of the Regge limit of the Virasoro-Shapiro amplitude describing tachyon-tachyon scattering (2.2), with one tachyon form factor $\mathcal{F}(q^2)$ replaced by $\mathcal{F}_{NN}(q^2)$. Nevertheless, the effective form factors are useful also for inelastic scattering ($N \neq N'$) and light targets. The reason is that (2.4) holds as long as we are in the Regge limit of high energy probes ($E \to \infty$) and fixed momentum transfer q^2, where strings are known to interact by exchanging the leading Regge trajectory as a whole.

2.2 Spatial distribution and random walks

Since we expect the random walk picture to be valid only for distances larger than the string scale, we will assume $q^2 \ll 1$. In this region the following simple expression for the form factor is valid

$$\mathcal{F}_{NN'}(q^2) \approx \frac{1}{2\pi} \int_0^{2\pi} d\xi \exp\left[i(N'-N)\xi - \frac{q^2}{2\beta} \xi(2\pi - \xi) \right] \tag{2.8}$$

with relative corrections of order $O(q^2)$.

A spatial distribution was obtained in [20] by Fourier transforming the *elastic* form factor

$$\rho(\vec{x}) = \frac{1}{(2\pi)^d} \int d^d q \, e^{i\vec{q}\cdot\vec{x}} \mathcal{F}_{NN}(q^2) = \frac{1}{2\pi} \left(\frac{\beta}{2\pi} \right)^{d/2} \int_0^{2\pi} d\xi h(\xi)^{-d/2} \exp\left(-\frac{\beta \vec{x}^{\,2}}{2h(\xi)} \right) , \tag{2.9}$$

where $h(\xi) \equiv \xi(2\pi - \xi)$ and $d \equiv D - 1$. Although $\rho(\vec{x})$ can not be evaluated analytically, it is obvious from (2.9) that

$$\int d^d x \rho(\vec{x}) = 1 \,. \tag{2.10}$$

We will now show that the density $\rho(\vec{x})$ admits a simple interpretation in terms of random walks. Consider a collection of discretized random paths $\vec{x}(\xi)$, where ξ is proportional to the number of steps, and such that $\vec{x}(0) = 0$. It is well known [23] that, in the continuum limit, the probability (density) for the walk to visit a point $\vec{x}$ is given by

$$P(\vec{x}, \xi) = (4\pi a\xi)^{-d/2} \exp\left(-\frac{\vec{x}^{\,2}}{4a\xi}\right) \tag{2.11}$$

where a is the diffusion coefficient if ξ is interpreted as 'time' for a brownian motion. Now, imagine a collection of *closed* random paths subject to the constraint $\vec{x}(0) = \vec{x}(2\pi) = 0$. Then, the conditional probability for the walk to visit a point $\vec{x}$ is given by

$$P_c(\vec{x}, \xi) = A \exp\left(-\frac{\vec{x}^{\,2}}{4a\xi}\right) \exp\left(-\frac{\vec{x}^{\,2}}{4a(2\pi - \xi)}\right) = A \exp\left(-\frac{\pi\vec{x}^{\,2}}{2a\xi(2\pi - \xi)}\right), \tag{2.12}$$

where the constant $A = [2a\xi(2\pi - \xi)]^{-d/2}$ normalizes the probability $P_c(\vec{x}, \xi)$ to 1. Then, the probability that the closed random walk visits a point $\vec{x}$ for *any* value of ξ is

$$P_c(\vec{x}) = \frac{1}{2\pi}\int_0^{2\pi} d\xi P_c(\vec{x}, \xi)\,. \tag{2.13}$$

But this is exactly $\rho(\vec{x})$ in (2.9), with $a = \pi/\beta$. In other words, $\rho(\vec{x})$ is created by a collection of closed random walks that begin and end at the origin! Note that we have recovered not just the second moment $\langle\vec{x}^{\,2}\rangle$, but the complete (closed) random walk distribution from the cross section. And since one can show, by using the exact formula for $\mathcal{F}_{NN'}(q^2)$ given in [20], that the corrections to (2.8) are rather small for $q^2 \lesssim 1$, we may conclude that the string 'looks' like a random walk even at resolutions close to the string scale.

There is, however, something rather puzzling in this picture. According to (2.13), all the random paths representing the ensemble of excited closed strings intersect at the origin. This is strange, since the origin is the center of mass, and there is no reason why the strings should go through it. Closely related to this is the fact, pointed out in [20], that for $1 \lesssim r \ll \sqrt{M}$, $\rho(r) \propto r^{2-d}$, i.e. the string distribution is singular at the origin. This would be quite natural for self-gravitating strings, but is rather surprising for the ensemble of *free* strings considered here. We will tackle these problems in the next section.

3. Inclusive scattering and structure functions

3.1 Density-density correlators

In order to better understand the physical meaning of the density $\rho(\vec{x}\,)$, it is convenient to consider the scattering cross section associated to the averaged rate (2.3). This is obtained by adding the appropriate phase factors and closed string coupling constant g_c. In the limit $M \gg E \gg 1$, where target recoil can be neglected and the the center of mass frame coincides with the target rest frame, the cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{g_c^4}{16EM^2}\frac{k'^{D-3}}{(2\pi)^{D-2}}\mathcal{R}(N, N', k, k')\,. \tag{3.1}$$

For elastic scattering ($N = N'$, $k = k' = E$) and $q^2 \ll 1$, we can use the approximate expression (2.7) for the rate, and the cross section becomes

$$\frac{d\sigma}{d\Omega} = \frac{g_c^4}{16(2\pi)^{D-2}} \left|EV(\vec{q})\mathcal{F}(\vec{q}^{\,2})\right|^2 k^{D-2}\,, \tag{3.2}$$

where we have defined a potential

$$V(\vec{q}) = M\frac{\mathcal{F}_{NN}(\vec{q}^{\,2})}{\vec{q}^{\,2}} \tag{3.3}$$

or, in terms of the Fourier transform (2.9), $\Delta V(\vec{x}) = -M\rho(\vec{x})$. Except for the presence of the tachyon form factor $\mathcal{F}(\vec{q}^{\,2})$ that reminds us that the probes are not point-like, (3.2) is just the formula for Born scattering[2] by a newtonian potential $V(\vec{x})$. In other words, the experimenter could interpret the elastic data as due to scattering by a target with mass distribution $M\rho(\vec{x})$.

For truly elastic scattering, where not only the energy but also the state of the target string is unchanged $\Phi_f = \Phi_i$, $\rho(\vec{x})$ would represent the density of the target string. In terms of Fourier transforms,

$$\mathcal{F}_{NN}(\vec{q}^{\,2}) = \langle\Phi_i|\rho_{st}(\vec{q})|\Phi_i\rangle\,, \tag{3.4}$$

where ρ_{st} is a suitably defined string density operator. However, we are considering an inclusive process where one averages over all initial states with fixed mass M and momentum p, and sums over final states of mass M' and momentum p'. Thus, even for 'elastic' ($M = M'$) scattering, the naïve relation (3.4) does not hold and $\rho(\vec{x})$, as given by (2.9), can *not* be understood as the actual density of the target string. Instead, the formula for the averaged inclusive rate (2.3) implies

$$\begin{aligned} |\mathcal{F}_{NN'}(q^2)|^2 &= \frac{1}{\mathcal{G}_c(N)} \sum_{\Phi_i|_N} \sum_{\Phi_f|_{N'}} |\langle\Phi_f|\rho_{st}(\vec{q})|\Phi_i\rangle|^2 \\ &= \frac{1}{\mathcal{G}_c(N)} \sum_{\Phi_i|_N} \sum_{\Phi_f|_{N'}} \langle\Phi_i|\rho_{st}(-\vec{q})|\Phi_f\rangle\langle\Phi_f|\rho_{st}(\vec{q})|\Phi_i\rangle\,. \end{aligned} \tag{3.5}$$

Note that we have dropped the elastic scattering constraint $N{=}N'$ as we are considering a general situation. In order to proceed, it is convenient to introduce the following function

$$\mathcal{C}_N(\vec{q},\omega) \equiv \sum_{N'} |\mathcal{F}_{NN'}(q^2)|^2 \delta(M' - M - \omega)\,, \tag{3.6}$$

where $\omega \equiv q^0$ is the energy transfer. Then, writing the delta function as a Fourier transform

$$\delta(M' - M - \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d(t-t')e^{i(M'-M-\omega)(t-t')} \tag{3.7}$$

[2] Note that the phase space factor k^{D-2} in (3.2) is appropriate for massless probes. In the more familiar case of nonrelativistic scattering with probes of mass m, the phase factor would be replaced by mk^{D-3}.

and using

$$\langle\Phi_f|\rho_{st}(\vec{q})|\Phi_i\rangle e^{i(M'-M)t} = \langle\Phi_f|\rho_{st}(\vec{q},t)|\Phi_i\rangle\,, \tag{3.8}$$

the double sum in (3.5) can be converted into a time dependent density-density correlator

$$\mathcal{C}_N(\vec{q},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d(t-t')e^{-i\omega(t-t')}\langle\rho_{st}(-\vec{q},t')\rho_{st}(\vec{q},t)\rangle_N\,, \tag{3.9}$$

where the ensemble average $\langle A\rangle_N$ of an operator is defined by

$$\langle A\rangle_N \equiv \frac{1}{\mathcal{G}_c(N)}\sum_{\Phi_i|_N}\langle\Phi_i|A|\Phi_i\rangle\,. \tag{3.10}$$

$\mathcal{C}_N(\vec{q},\omega)$ is thus the dynamical *structure function*, i.e., the Fourier transform of the time-dependent density-density correlator for the ensemble of target strings. Inverting the Fourier transforms gives

$$\int d^dx\langle\rho_{st}(\vec{x}+\Delta\vec{x},t+\Delta t)\rho_{st}(\vec{x},t)\rangle_N = \frac{1}{(2\pi)^d}\int d^dq\,d\omega\,e^{i(\vec{q}\cdot\Delta\vec{x}-\omega\Delta t)}\,\mathcal{C}_N(\vec{q},\omega)\,, \tag{3.11}$$

where $d = D-1$ and $(\Delta\vec{x},\Delta t)$ is an arbitrary, not necessarily small, space-time displacement. Finally, using (3.6) yields the correct relation between the string density ρ_{st} and the form factor $\mathcal{F}_{NN'}$

$$\int d^dx\langle\rho_{st}(\vec{x}+\Delta\vec{x},\Delta t)\rho_{st}(\vec{x},0)\rangle_N = \sum_{N'}\frac{1}{(2\pi)^d}\int d^dq\,e^{i(\vec{q}\cdot\Delta\vec{x}-\omega\Delta t)}\,|\mathcal{F}_{NN'}(q^2)|^2\,, \tag{3.12}$$

where $\omega = M'-M$ and we have set $t=0$ since, according to (3.11), the correlator depends only on the time difference Δt. Thus the (partially integrated) density-density correlator is the Fourier transform of the square of the inelastic form factor. Actually, it is a series rather than a Fourier transform in ω, implying periodicity in time with the period T fixed by the mass shell condition

$$\omega = M'-M \simeq \frac{(N'-N)}{M} \implies T = 2\pi M\,. \tag{3.13}$$

3.2 Probabilities from correlators

The Fourier transform of the form factor is easily obtained by noting that the spatial part is a simple gaussian and (2.8) is already given as a Fourier coefficient with $i(N'-N)\xi = i\omega t$. The result is

$$\sum_{N'}\frac{1}{(2\pi)^d}\int d^dq\,e^{i(\vec{q}\cdot\vec{x}-\omega t)}\,\mathcal{F}_{NN'}(q^2) \simeq (\pi f(t))^{-d/2}\exp\left(-\frac{\vec{x}^{\,2}}{f(t)}\right) \equiv P_c(\vec{x},t) \tag{3.14}$$

where $f(t)$ is a periodic function that, for $0<t<T=2\pi M$, is given by

$$f(t) = \frac{4T_H}{M}t\,(T-t)\,. \tag{3.15}$$

This shows that the Fourier transform of the inelastic form factor is just the conditional probability (2.12) with $t = M\xi$. In evaluating (3.14), the approximation $q^2 = \vec{q}^{\,2} - \omega^2 \simeq \vec{q}^{\,2}$ has been made. This is justified by noting that, at low momentum transfer, (2.8) is non-negligible only for $\omega \lesssim \vec{q}^{\,2}$. By (3.12), the density-density correlator can be written as a space-time convolution of conditional probabilities

$$\int d^d x \langle \rho_{st}(\vec{x} + \Delta\vec{x}\,, \Delta t)\rho_{st}(\vec{x}\,, 0)\rangle_N = \frac{1}{T}\int_0^T dt' \int d^d x'\, P_c(\vec{x}', t')\, P_c(\Delta\vec{x} - \vec{x}', \Delta t - t')\,. \quad (3.16)$$

The physical meaning of this equation, which is the main result of this section, can be clarified by noting that the density-density correlator

$$\langle \rho_{st}(\vec{x}\, + \Delta\vec{x}\,, \Delta t)\rho_{st}(\vec{x}\,, 0)\rangle_N$$

is proportional to the conditional probability to detect a string (with center of mass at the origin) at point $\vec{x} + \Delta\vec{x}$ for $t = \Delta t$, assuming the string has been previously detected at point $\vec{x}$ for $t = 0$. This probability is *not* translational invariant and depends not only on the distance $\Delta\vec{x}$, but also on $\vec{x}$. This is so because the ensemble includes all states at mass level N with fixed total momentum p, i.e., we average over oscillator states, but not over center of mass wave functions. In position space, our target is a highly excited string with center of mass at the origin, and the correlator has to go to zero as its arguments move away from it.

Integrating over $\vec{x}$ is obviously equivalent to dropping the fixed center of mass constraint. In other words, the integrated correlator in the l.h.s. of (3.16) is simply proportional to the probability $P_{st}(\Delta\vec{x}\,, \Delta t)$ that *any* string of mass M that has been previously detected at a point is detected again after a time Δt at a distance $\Delta\vec{x}$, irrespective of its center of mass position. Actually, the constant of proportionality is 1 and (3.16) can be rewritten as

$$P_{st}(\Delta\vec{x}\,, \Delta t) = \frac{1}{T}\int_0^T dt' \int d^d x'\, P_c(\vec{x}', t')\, P_c(\Delta\vec{x} - \vec{x}', \Delta t - t') \quad (3.17)$$

since, as one can easily verify, P_{st} is correctly normalized

$$\int d^d x P_{st}(\vec{x}\,, t) = 1\,. \quad (3.18)$$

The periodicity in time mentioned above means that a configuration of the ensemble of target strings repeats itself after a time $2\pi M$:

$$P_{st}(\Delta\vec{x}\,, \Delta t + 2\pi M) = P_{st}(\Delta\vec{x}\,, \Delta t)\,. \quad (3.19)$$

For $\Delta t = 0$, doing the x' integral in (3.17) gives the probability for simultaneous detection of the string at two points separated by $\Delta\vec{x}$. The result is

$$P_{st}(\Delta\vec{x}\,, 0) = \frac{1}{T}\int_0^T dt' P_c\left(\frac{\Delta\vec{x}}{\sqrt{2}}, t'\right) = \frac{1}{4\pi M}\int_0^{4\pi M} d\sigma P_c\left(\frac{\Delta\vec{x}}{\sqrt{2}}, \frac{\sigma}{2}\right) \quad (3.20)$$

where we have made the change of variables $\sigma = 2t'$. Note that

$$P_c\left(\frac{\Delta\vec{x}}{\sqrt{2}}, \frac{\sigma}{2}\right) \propto \exp\left(- \frac{M}{2T_H}\frac{\Delta\vec{x}^{\,2}}{\sigma(4\pi M - \sigma)}\right) \quad (3.21)$$

has the same form as (2.12) and represents the probability that any closed path of length $4\pi M$ and unconstrained center of mass goes simultaneously through two points separated by an interval $\Delta\vec{x}$. Thus, at any given time the ensemble of strings looks like a collection of closed random walks parametrized by $0 \leq \sigma \leq 4\pi M$.

Although (3.20) is formally similar to (2.9) or (2.13), the interpretation is quite different. While in (2.9) or (2.13) $\vec{x}$ is the distance to a *fixed* point, the center of mass, $\Delta\vec{x}$ in (3.20) is the distance to *any* point where the string is simultaneously detected. Thus, recognizing that the string density ρ_{st} is related to $\mathcal{F}_{NN'}$ by (3.12), rather than by (2.9), avoids the problematic conclusion that all the strings go through a common point and solves the puzzle mentioned at the end of last section.

Moreover, (3.17) contains more than purely static information. Indeed, (3.17) expresses the time evolution of the ensemble of highly excited strings as a convolution of (closed) brownian motions. Concretely, the probability that any string goes through two different points separated by a space-time interval $(\Delta\vec{x}, \Delta t)$ involves two closed random paths: one going from the initial point to any space-time point within a time period $T = 2\pi M$, and another going from this intermediate point to the final point. In what follows, we will show that this somewhat surprising structure is quite natural from a semiclassical point of view.

3.3 Semiclassical interpretation

Any classical closed string solution [21, 22] can be written as the sum of two arbitrary functions subject to the constraints $(\partial_+ X_+)^2 = (\partial_- X_-)^2 = 0$,

$$X^\mu(\tau,\sigma) = X^\mu_+(\sigma_+) + X^\mu_-(\sigma_-)\,, \qquad \sigma_\pm = \frac{1}{2}(\tau \pm \sigma) \tag{3.22}$$

where $X^\mu(\tau, \sigma + T_\sigma) = X^\mu(\tau,\sigma)$. It is easier to work in the temporal gauge defined by

$$X^0 = \tau\,, \qquad X^0_\pm = \sigma_\pm\,. \tag{3.23}$$

The period T_σ is determined by noting that the total energy-momentum of the string is given by the Noether current

$$P^\mu = \frac{1}{4\pi}\int_0^{T_\sigma} d\sigma\, \partial_\tau X^\mu\,. \tag{3.24}$$

This formula is valid for a string tension $(2\pi\alpha')^{-1} = (4\pi)^{-1}$. In the center of mass frame, $P^0 = M$ and (3.24) implies $T_\sigma = 2T = 4\pi M$, and $\vec{X}_\pm(\sigma_\pm + 2\pi M) = \vec{X}_\pm(\sigma_\pm)$. The constraints become

$$(\partial_+ \vec{X}_+)^2 = (\partial_- \vec{X}_-)^2 = 1 \tag{3.25}$$

and we see that each classical solution is represented by two arbitrary d-dimensional closed curves of length $2\pi M$. On each curve, $\sigma_\pm$ plays the role of length parameter.

Now, the classical probability for a string to go through two points separated by the interval $(\Delta\vec{x}, \Delta t)$ will be proportional to the 'number' of pairs of d-dimensional closed curves of length $2\pi M$ that go through the first point, that we may choose as the origin,

$$\vec{X}_\pm(0) = \vec{X}_\pm(2\pi M) = 0\,, \tag{3.26}$$

and such that

$$\vec{X}_+(\sigma_+) + \vec{X}_-(\sigma_-) = \Delta\vec{x}\,, \qquad \sigma_\pm = \frac{1}{2}(\Delta t \pm \sigma) \tag{3.27}$$

for any value of the parameter $\sigma \in [0, 2T]$. If one assumes that the number of curves satisfying $\vec{X}_\pm(\sigma_\pm) = \vec{x}$ is given by $P_c(\vec{x}, \sigma_\pm)$, then the classical probability is necessarily given by

$$P_{st}(\Delta\vec{x}\,, \Delta t) = \frac{1}{2T}\int_0^{2T} d\sigma \int d^dX_+\, P_c\left(\vec{X}_+, \frac{1}{2}(\Delta t + \sigma)\right) P_c\left(\Delta\vec{x} - \vec{X}_+, \frac{1}{2}(\Delta t - \sigma)\right). \tag{3.28}$$

With the substitutions $\sigma = 2t' - \Delta t$ and $\vec{X}_+ = \vec{x}\,'$, this is exactly (3.17).

This argument is heuristic because the 'number of curves' is always infinite, unless a cut-off is introduced. In fact, this cut-off is forced upon us by the breakdown of the classical description. A string can be viewed as a collection of oscillators α_n^μ, that may be described classically only for large values of occupation numbers N_n^μ. The mass level is given by

$$\frac{1}{2}M^2 + 1 = N = \sum_{n=1}^{\infty}\sum_{\mu=1}^{D} n\, N_n^\mu\,. \tag{3.29}$$

Then, for large N,

$$\langle N_n^\mu \rangle \sim \frac{\sqrt{N}}{n} \sim \frac{M}{n} \tag{3.30}$$

and the oscillators admit a classical description only for n less that the mass of the string in string units. Since we are considering curves of length $2\pi M$, the wavelength corresponding to the oscillator α_n^μ is $\lambda \sim M/n$, and the breakdown takes place for $\lambda \sim 1$, i.e., at the string scale. Thus, each curve should be discretized as a collection of $\sim M$ pieces or string bits. Note that in this argument the string bits appear, not as fundamental objects, but as an effective way of describing a highly excited string semiclassically.

Now, counting curves with a cut-off ϵ is equivalent to counting random walks, and in the continuum limit the probability that an ordinary random walk goes from the origin to $\vec{x}$ is [23]

$$P(\vec{x}, \sigma) \sim \exp\left(-\frac{b\vec{x}^{\,2}}{\epsilon\sigma}\right) \tag{3.31}$$

as long as σ is a length parameter. The constant b depends both on the dimension d and the details of the discretization, but is of order one. For a closed random walk of length T

$$P_c(\vec{x}, \sigma) \sim \exp\left(-\frac{b\vec{x}^{\,2}}{\epsilon\sigma}\right) \exp\left(-\frac{b\vec{x}^{\,2}}{\epsilon(T-\sigma)}\right) \sim \exp\left(-\frac{bT\vec{x}^{\,2}}{\epsilon\sigma(T-\sigma)}\right). \tag{3.32}$$

This is (3.14) for $\epsilon \sim 1$, up to a constant of order one in the exponent.

4. Decoherence and singular backgrounds

In this section we will try to understand the differences between the string density $\rho_{st}(\vec{x}, t)$, related to the form factor by (3.12), and the density $\rho(\vec{x})$ given by (2.9). As argued at the beginning of section 3, the fact that the effective form factor is obtained from an averaged inclusive cross section implies that $\rho(\vec{x})$ can *not* represent the actual string density of the target. They are in fact very different functions. As mentioned above, $\rho(\vec{x})$ is singular at the origin, with a divergence $\rho(r) \sim r^{2-d}$. On the other hand, according to the analysis in section 3, an instantaneous picture of the target would show an ensemble of closed random walks, all of them with center of mass at the origin. It is intuitively obvious that the associated density should be perfectly regular.

This can be shown explicitly. Although (3.20) is missing the fixed center of mass constraint due to the $\vec{x}$-integral in (3.12), this condition is easily restored with the use of a path integral formalism. The details of this computation are presented in the appendix, where it is shown that

$$\rho_{st}\,(\vec{x},t) = B \exp\left(-\frac{3\vec{x}^{\,2}}{8\pi^2 M T_H}\right) \tag{4.1}$$

with $B = (8\pi^3 M T_H/3)^{-d/2}$. Thus ρ_{st} is independent of t, as one would expect of an ensemble of stationary states, and regular. The mean square radius computed from it

$$\langle\,\vec{x}^{\,2}\rangle = \frac{4\pi^2 d}{3} T_H M \tag{4.2}$$

agrees with the result obtained by oscillator methods [2, 3].

The analysis at the beginning of section 3 shows that the elastic cross section could also be interpreted as due to potential scattering with $\rho(\vec{x})$ acting as a source, i.e., $\Delta V(\vec{x}) = -M\rho(\vec{x})$. However, this interpretation is incomplete because of the the occurrence of inelastic processes that can not be accounted for by a time-independent potential. We are thus led to consider the inelastic generalization of the cross section (3.2). Taking the $q^2 \ll 1$ limit of (2.4) and using (3.1) yields

$$\frac{d\sigma}{d\Omega} = \frac{g_c^4}{16(2\pi)^{D-2}} \left|V(\vec{q},\omega)\mathcal{F}_{EE'}(\vec{q}^{\,2})\right|^2 EE'\,k'^{D-2}\,, \tag{4.3}$$

where we have defined

$$V(\vec{q},\omega) = M\frac{\mathcal{F}_{NN'}(\vec{q}^{\,2})}{\vec{q}^{\,2}} \tag{4.4}$$

with $\omega = E - E' = M' - M \simeq \sqrt{2N'} - \sqrt{2N}$. $\mathcal{F}_{EE'}$ can be interpreted as an inelastic tachyon form factor and is given by

$$\mathcal{F}_{EE'}(q^2) = e^{-\frac{q^2}{2}\log EE'}\,. \tag{4.5}$$

Thus, the main differences between (4.3) and the elastic cross-section (3.2) are the replacements

$$|E\mathcal{F}(\vec{q}^{\,2})|^2 \longrightarrow EE'|\mathcal{F}_{EE'}(\vec{q}^{\,2})|^2\,, \qquad V(\vec{q}) \longrightarrow V(\vec{q},\omega)\,. \tag{4.6}$$

The first one is quite trivial and reflects the fact that the incoming and outgoing probes have different energies. The second replacement means that, in order to have inelastic processes in potential scattering, the potential has to be time-dependent. The Fourier transform of (4.4), $\Delta V(\vec{x}\,,t) = -M\rho_{\text{eff}}(\vec{x}\,,t)$, implies that the potential is created by the time-dependent effective density

$$\rho_{\rm eff}(\vec{x}\,,t) = \sum_{N'} \frac{1}{(2\pi)^d} \int d^d q\, e^{i(\vec{q}\cdot\vec{x}-\omega t)}\, \mathcal{F}_{NN'}(\vec{q}^{\,2}) = (\pi f(t))^{-\frac{d}{2}} \exp\left(-\frac{\vec{x}^{\,2}}{f(t)}\right), \tag{4.7}$$

where $f(t)$ is the periodic function given by (3.15). Then note that, according to (2.9), $\rho(\vec{x})$ is simply the time average of $\rho_{\rm eff}(\vec{x},t)$

$$\rho\,(\vec{x}) = \frac{1}{T}\int_0^T dt\, \rho_{\rm eff}(\vec{x}\,,t)\,. \tag{4.8}$$

The qualitative features of $\rho_{\rm eff}(\vec{x},t)$ and the associated potential $V(\vec{x},t)$ are rather suggestive. The mean square radius of the distribution is

$$\langle\,\vec{x}^{\,2}\rangle(t) = \frac{d}{2}f(t) = \frac{2\,T_H d}{M} t\,(2\pi M - t) \tag{4.9}$$

and goes to zero periodically for $t = n(2\pi M)$, $n\in\mathbb{Z}$, where the density collapses to a delta function. Note that it takes a time $2\pi M$ for the radius to grow from zero to the maximum value $r_{\rm max} = \pi\sqrt{2MT_H d}$ and then collapse back to zero. Thus, for large M, the potential is slowly varying. This explains the absence of retarded effects in the relation $V(\vec{x}\,,t) = -M\rho_{\rm eff}(\vec{x}\,,t)$ and justifies the use of an instantaneous newtonian potential.[3]

For $r > r_{\rm max}$ the density goes to zero exponentially, the potential approaches that of a point mass and tends to become static. Thus we have an outer, approximately static region, and an inner dynamical region where the potential is time-dependent and corresponds to a spherically symmetric mass distribution that periodically collapses and bounces back. Note that this is vaguely reminiscent of the maximally extended Schwarzchild solution [24]: Above the horizon the geometry is static but, inside the gravitational radius, r is a time-like coordinate and the geometry becomes time-dependent, evolving from a 'big bang' to a 'big crunch' in a time proportional — in four dimensions — to the mass of the black hole.

Thus, from the same scattering cross sections we can get two entirely different portraits of the scatterer. If we are aware of the decoherence implied by averaging and summing over enormous numbers of string states in (2.3), we will use (3.12) to extract information on density-density correlators, ending up with a description of the target in terms of random walks. We will find that the 'actual' density of the string ρ_{st}, given by (4.1), is time-independent and regular. On the other hand, if we assume that the scattering is caused by a *classical* background, we will conclude that the corresponding 'apparent' or effective density $\rho_{\rm eff}$ is both time-dependent and singular. We may say that, at least in the example considered here, singularities arise not as a result of strong gravitational self-interactions — after all, the target is a highly excited *free* string — but as a byproduct of the decoherence implicit in effectively describing the string degrees of freedom as a classical background.

We would like to close this section by pointing out that $\rho_{\rm eff}$, given by (4.7), is not unique. The reason is that the interaction rate (2.4) determines the effective form factor

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[3]Technically, this is related to the approximation $q^2 = \vec{q}^{\,2} - \omega^2 \simeq \vec{q}^{\,2}$ used in (4.3) and (4.4). See also comment after (3.15).

only up to an arbitrary phase. Imposing time-reversal symmetry leads uniquely to the real form factor (2.8) and to the time-symmetric effective density (4.7). Dropping this constraint amounts to a redefinition

$$\mathcal{F}_{NN'}(q^2) \to \mathcal{F}_{NN'}(q^2)\, e^{\,i\varphi(\vec{q},\omega)}\,, \tag{4.10}$$

where reality of $\rho_{\rm eff}(\vec{x},t)$ requires $\varphi(\vec{q},-\omega) = -\varphi(\vec{q},\omega)$. However, one can easily check that $\varphi(\vec{q},0) = 0$ implies that the time averaged density $\rho(\vec{x})$ given by (4.8) is invariant under (4.10). As $\rho(\vec{x})$ is singular at the origin, so has to be the effective density $\rho_{\rm eff}(\vec{x},t)$, although the singularity may be more spread in time if the assumption of time-reversal symmetry is relaxed.

5. Discussion

In this paper we have confirmed the old suggestion [1, 2, 3] that highly excited strings behave like random walks. We have done this by extracting — from the appropriate averaged inclusive cross sections — probabilities for joint detection of the target string at two different space-time points. These probabilities can be written as a convolution (3.17) of closed random walks and provide, not just a static picture of the ensemble of excited closed strings, but also a description of its time evolution.

The form of the joint probability (3.17) can also be inferred from the semiclassical argument presented at the end of section 3. Nevertheless, it is satisfying to see it emerge from the full-fledged string computation in [20] after a set of well controlled approximations.[4] In fact, one could turn the argument around and use semiclassical information to obtain effective form factors and inclusive cross sections for highly excited strings, without ever doing a detailed string computation. Indeed, inverting (3.12) and writing the partially integrated correlator in terms of probabilities yields

$$|\mathcal{F}_{NN'}(q^2)|^2 = \frac{1}{T}\int_0^T dt \int d^dx\, e^{-i(\vec{q}\cdot\vec{x}-\omega t)}\, P_{st}(\vec{x}\,,t)\,. \tag{5.1}$$

This formula is useful because the probability in the r.h.s. can be written directly for any state admitting a semiclassical interpretation. Such is the case, for instance, for the long lived string states considered in [25]–[29]. The point is that (3.12), and therefore (5.1), are valid not just for the mixed state represented by the ensemble average (3.10), but for any initial target state, mixed or pure. Indeed, one can easily check that the argument leading to (3.12) goes through as long as the ensemble average (3.10) is replaced by

$$\langle A\rangle \equiv {\rm Tr}(\hat{\rho}A)\,, \tag{5.2}$$

where $\hat{\rho}$ is the appropriate density matrix — not to be confused with the spatial densities considered in this paper! In other words, as long as one considers inclusive processes where

[4]The excellent agreement between the quantum string computation and the semiclassical argument indicates that typical highly excited strings admit a semiclassical description almost down to the string scale. Non-typical states may, or may not, admit such a description.

the *final* state of the target is not measured, one ends up with a formula relating the cross section to an averaged density-density correlator. Considering different initial states only affects the type of average.

Nevertheless, one should be aware that (5.1) is valid only for (relatively) small momentum transfer. This is not surprising, since any geometric description of string states is expected to break down at the string scale. Less obvious is the requirement of high energy probes, which is necessary for the validity of the factorization property (2.4) of the averaged rate [20]. The reason may be that, in the Regge limit of high energy and fixed q^2, the scattering is dominated by t-channel exchange, whereas at lower energies s-channel capture and re-emission as thermal radiation probably plays an important role.

One possible generalization of the results presented here is the extension to superstring inclusive cross sections. There one should face the problem of defining what is meant by a 'super random walk'. As far as we know, no detailed generalization of the random walk model has been worked out for the superstring. One obvious problem is the fact that the occupation numbers for fermionic oscillators can never be large, and it is not clear what a semiclassical description means in that case. One clue may be provided by the computation of the massless emission rate for a very long lived superstring state in [29]. The correct rate for the dominant decay channel is obtained by a classical computation that includes only bosonic degrees of freedom. It would also be interesting to use the random walk picture developed in this paper to compute the decay properties of typical highly excited strings. The results could be compared with those obtained by oscillator methods in [30, 31] for the bosonic string, and in [29, 32] for the superstring.

Throughout this paper we have considered two different densities. On the one hand we have the string density ρ_{st} that represents the actual string distribution and is regular and time-independent — even though its two point correlators are time-dependent. On the other hand is the effective density ρ_{eff}, that turns out to be both singular and time-dependent. The effective density ρ_{eff} acts as the source for the classical potential that reproduces the inclusive cross sections in the Born approximation, but does not describe the actual string distribution. By the same token, the string density ρ_{st} can not be considered as the source for the classical background.

In this regard, we would like to stress the role played by decoherence. Indeed, both the random walk picture and the form of the effective density (4.7) are valid only for the ensemble considered here, i.e., for the density matrix

$$\hat{\rho} = \frac{1}{\mathcal{G}_c} \sum_{\Phi_i|N} |\Phi_i\rangle\langle\Phi_i| \,. \tag{5.3}$$

Other initial states, mixed or pure, will give rise to different classical backgrounds. This is reminiscent of the situation in the D-brane approach to black holes. As stressed in [33, 34], in order to get the correct black hole properties one has to consider a decoherent ensemble of all degenerate D-brane configurations. Less generic configurations in the D-brane side give rise to geometries that cannot be interpreted as black holes [35]–[37]. On the other hand, it is the decoherence associated with not measuring the *final* state of the target in an inclusive cross section that seems to be responsible for the striking differences between ρ_{st} and ρ_{eff} in our case.

In the D-brane approach one usually computes physical properties such as entropies or grey-body factors at weak coupling, and compares the results with those obtained by considering a strongly coupled general relativity solution. Even though singularities and horizons appear only on the gravity side, one usually assumes that, as the coupling is increased, the microscopic string configuration undergoes a collapse and becomes the source for the black hole geometry [6]–[9]. The example studied in this paper suggests a different possibility, more in line with [18]: In our computations, both the string system and the classical background are at weak coupling, and yet the corresponding densities ρ_{st} and ρ_{eff} are very different. As mentioned above, one is static and regular, and can not even be considered as the source for the background potential; the other is singular and describes a sequence of collapses and 'big-bangs'. So, maybe the strings never collapse, and collapse and 'no-collapse' are just complementary views of the same reality.

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A. Computation of the string density

In this appendix we formulate the (fixed time) sum over closed random walks as a euclidean path integral, and use it to compute the string density ρ_{st}. The probability that a string with free center of mass goes *simultaneously* through two points separated by an interval $\Delta\vec{x}$ is given by (3.21). Writing

$$\exp\left(-\frac{M}{2T_H}\frac{\Delta\vec{x}^{\,2}}{\sigma(4\pi M-\sigma)}\right)=\exp\left(-\frac{1}{8\pi T_H}\frac{\Delta\vec{x}^{\,2}}{\sigma}\right)\exp\left(-\frac{1}{8\pi T_H}\frac{\Delta\vec{x}^{\,2}}{\sigma(4\pi M-\sigma)}\right) \tag{A.1}$$

suggests the following form for the path integral

$$\mathcal{I}_0=\int\mathcal{D}\vec{X}\,e^{-S_0[\vec{X}]}\,,\qquad S_0[\vec{X}]=\frac{1}{8\pi T_H}\int_0^{4\pi M}d\sigma\left(\frac{d\vec{X}}{d\sigma}\right)^2 \tag{A.2}$$

subject to the periodicity condition $\vec{X}(4\pi M)=\vec{X}(0)$. Indeed, this is just the generalization of (A.1) to a path that has to go through an infinite number of points in the continuum limit.

As a check of (A.2), consider the sum over all periodic paths that go through two points separated by $\Delta\vec{x}$. As $S_0[\vec{X}]$ is quadratic, it is enough to compute the classical solution to $\vec{X}'' = 0$, subject to the constraints $\vec{X}(0) = \vec{X}(4\pi M) = 0$ and $\vec{X}(\sigma) = \Delta\vec{x}$. This is given by

$$\vec{X}_{cl}\,(\sigma') = \begin{cases} \left(\dfrac{\sigma'}{\sigma}\right)\Delta\vec{x} & 0 \le \sigma' \le \sigma \\ \left(\dfrac{\sigma' - 4\pi M}{\sigma - 4\pi M}\right)\Delta\vec{x} & \sigma < \sigma' \le 4\pi M\,. \end{cases} \tag{A.3}$$

The action for this classical path is

$$S_0[\vec{X}_{cl}] = \frac{1}{8\pi T_H}\int_0^{4\pi M} d\sigma'\vec{X}_{cl}^{\,\prime 2} = \frac{M}{2T_H}\frac{\Delta\vec{x}^{\,2}}{\sigma(4\pi M - \sigma)} \tag{A.4}$$

which agrees with the exponent in (A.1).

In order to compute ρ_{st} for an ensemble of strings with center of mass at the origin, the action has to be modified so that the path integral incorporates this condition. This is achieved by defining

$$\mathcal{I}_1 = \int \mathcal{D}\vec{X}\, e^{-S_1[\vec{X}]}\,, \qquad S_1[\vec{X}] = S_0[\vec{X}] + \frac{1}{4\pi T_H}\int_0^{4\pi M} d\sigma\,\vec{\mu}\cdot\vec{X}\,(\sigma)\,, \tag{A.5}$$

where $\vec{\mu}$ is a σ-independent Lagrange multiplier that enforces the constraint $\vec{X}_{cm} = 0$. Now, consider the sum over all periodic paths — with center of mass at the origin — that go through a point $\vec{x}$. We can use invariance under translations in σ to set $\vec{X}(0) = \vec{x}$. As before, the path integral is determined by the classical solution to the equation of motion $\vec{X}\,'' = \vec{\mu}$,

$$\vec{X}_{cl}\,(\sigma) = \vec{a} + \vec{b}\,\sigma + \frac{1}{2}\,\vec{\mu}\,\sigma^2\,. \tag{A.6}$$

Imposing $\vec{X}_{cl}(0) = \vec{X}_{cl}(4\pi M) = \vec{x}$ yields $\vec{a} = \vec{x}$, $\vec{b} = -2\pi M\vec{\mu}$. Then,

$$\int_0^{4\pi M} d\sigma\,\vec{X}_{cl}\,(\sigma) = 0 \implies \vec{\mu} = \frac{3\vec{x}}{4\pi^2 M^2} \tag{A.7}$$

and

$$S_1[\vec{X}_{cl}] = \frac{1}{8\pi T_H}\int_0^{4\pi M} d\sigma\vec{X}_{cl}^{\,\prime 2} = \frac{3\vec{x}^{\,2}}{8\pi^2 M T_H}\,. \tag{A.8}$$

This, together with the normalization condition, gives (4.1) uniquely.

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